Grid-Induced Numerical Errors for Shear Stresses and Essential Flow Variables in a Ventricular Assist Device: Crucial for Blood Damage Prediction?

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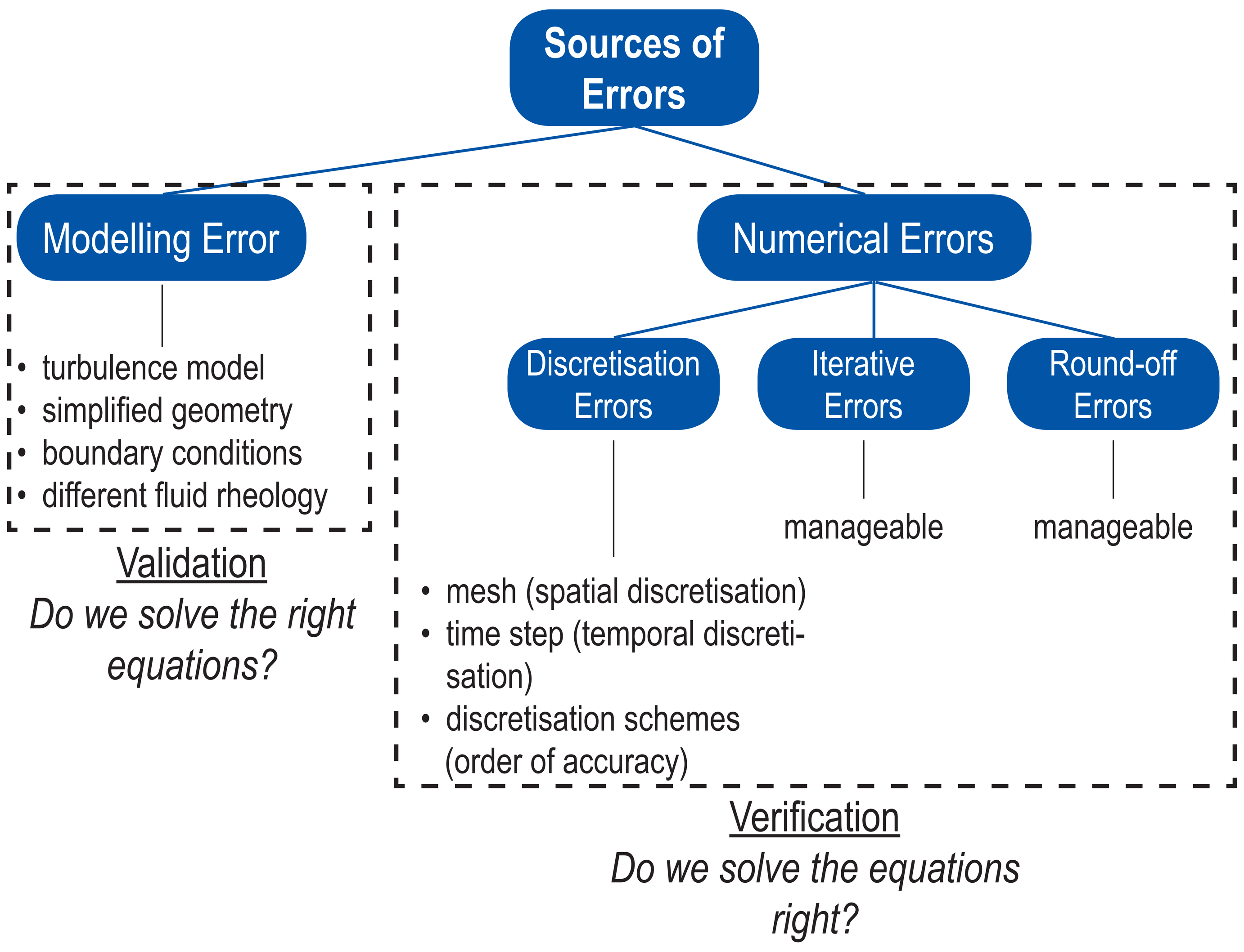
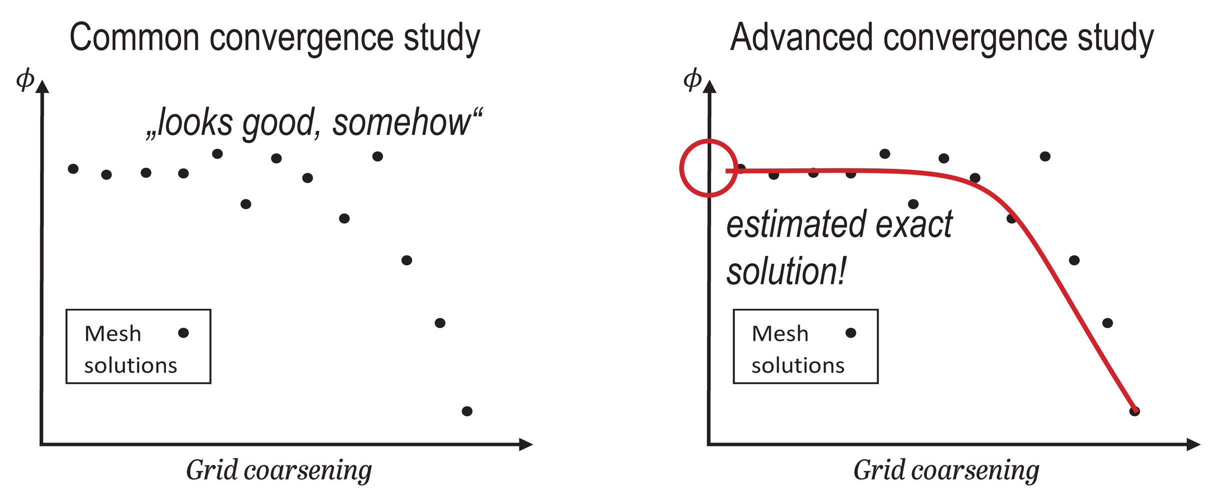
Motivation

Computational Fluid Dynamics (CFD) have become an indispensable tool for the development and optimisation of Ventricular Assist Devices (VADs) and other blood-contacting devices. Results can be generated very fast in comparison to time-consuming and cost-intensive experiments. But in contrast to experimental results, the quantification of errors is difficult with numerical solutions.



Quantifying the Error

The most-used method for verifying numerical solutions is a mesh convergence study. Background: With ongoing refinement of the computational mesh, the solution should converge to a grid-independent state, i.e. the asymptotical range where further decrease of cell size h does not result in a change of variables. How fast the solution approaches this state depends on the order of used discretisation schemes and other factors.



$$error = \phi_i - \phi_0 = \alpha h_i^p$$

solution on i-th mesh — exact solution — continuous fit

The unknown constant α, order of convergence p and exact solution φ_0 are determined by the minimum of the function

$$S_{RE}(\phi_0, \alpha, p) = \sqrt{\sum_{i=1}^{n_g} w_i \left(\phi_i - (\phi_0 + \alpha h_i^p)\right)^2} \qquad \frac{\partial S_{RE}}{\partial \phi_0} = 0, \quad \frac{\partial S_{RE}}{\partial \alpha} = 0, \quad \frac{\partial S_{RE}}{\partial p} = 0$$

This leads to a system of non-linear equations. The uncertainties (95%-confidence interval) for all meshes can be calculated by

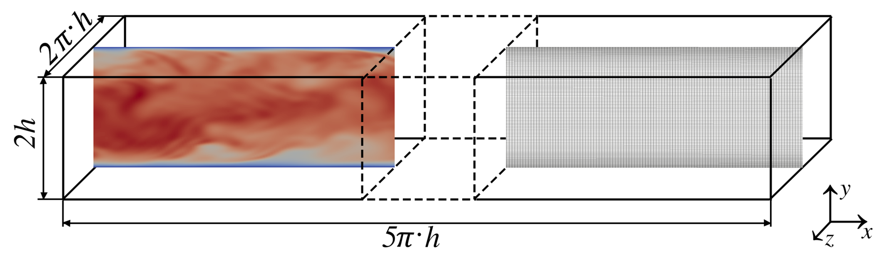
$$U_\phi(\phi_i) = F_s \epsilon_\phi(\phi_i) + \sigma + |\phi_i - \phi_{fit}|$$

safety factor (1.25) — error — standard deviation (between fit and CFD) — absolute deviation

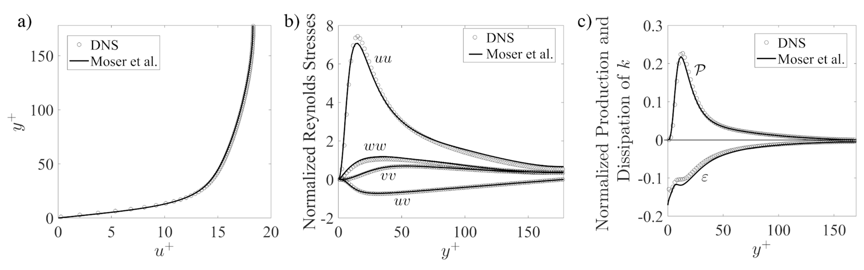
Method Confirmation

To test the proper function of the uncertainty method, we applied it on a test case: a turbulent channel flow computed by a direct numerical simulation (DNS).

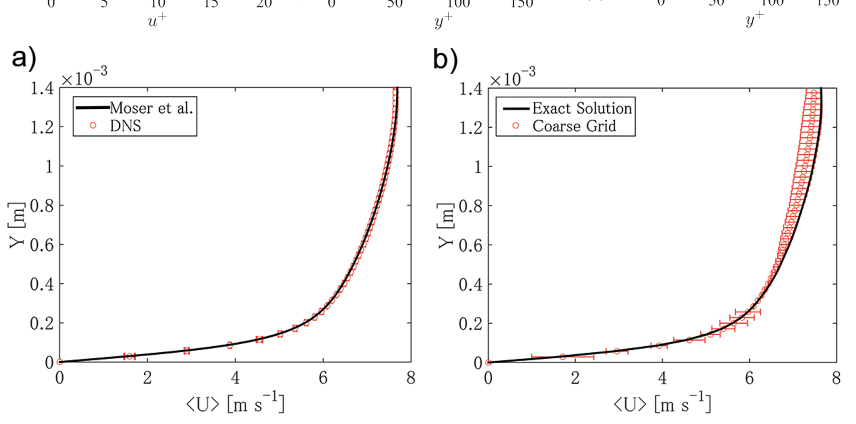
Computational setup. Left: Snapshot of velocity in cut-plane. Right: DNS-mesh.



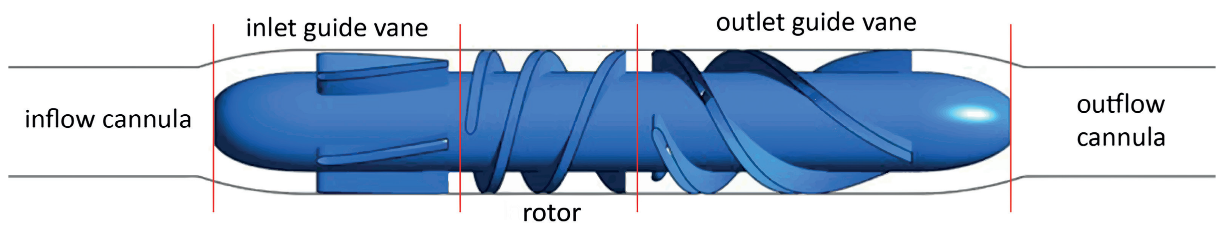
Comparison of results (f.l.t.r. mean velocity profile, Reynolds stresses, TKE budget) with literature data.



Left: Velocity profile and uncertainties with finest mesh. Right: Same data obtained on coarsest mesh. It can be seen, that the calculated uncertainties fit well with the exact solution from the literature.

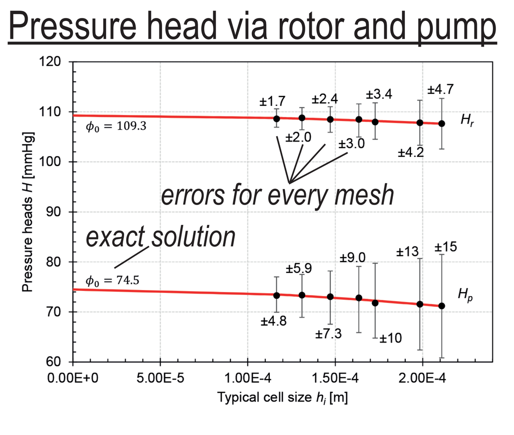


Simulation of VAD Flow



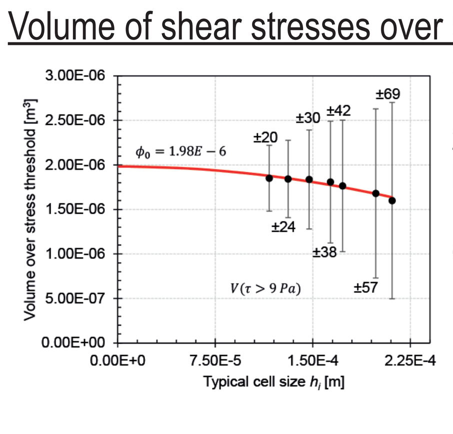
- Q = 4.5 l/min @ 7900 rpm
- k-ω-SST turbulence model
- seven meshes from 3.3M to 19M elements
- y^+ around 1 and smaller
- second-order schemes in time and space
- time-averaging of variables over ten revolutions
- only resolved stresses are considered

Pressure head via rotor and pump



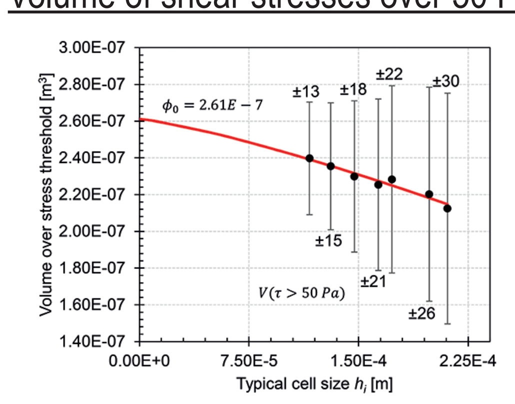
Error for finest mesh:
Pump: 1.7 %
Rotor: 4.8 %
For coarsest mesh:
Pump: 4.7 %
Rotor: 15 %

Volume of shear stresses over 9 Pa



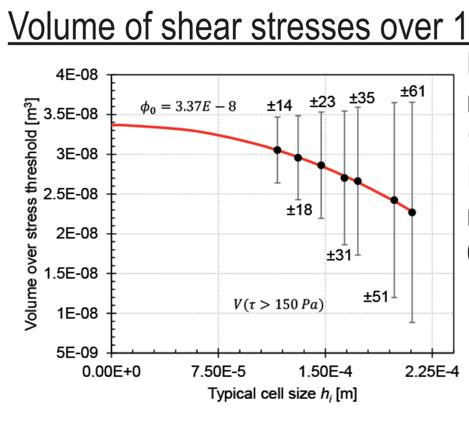
Error for finest mesh:
20 %
For coarsest mesh:
69 %

Volume of shear stresses over 50 Pa



Error for finest mesh:
13 %
For coarsest mesh:
30 %

Volume of shear stresses over 150 Pa



Error for finest mesh:
14 %
For coarsest mesh:
61 %

Conclusion & Limitations

- Uncertainties on the finest mesh for first order quantities (pressure head) are sufficiently low, but very high for second order quantities (volumes above shear stress threshold)

Shear stresses have higher grid-dependency!

- Uncertainty calculation only possible for RANS/URANS computations
- Works best with hexahedral meshes
- Needs consistently coarsened meshes
- Assumption: other error sources constant over meshes

More can be found in: Konnigk et al. J. Verif. Valid. Uncert. 2018, 3(4); Eça and Hoekstra. J. Comput. Phys. 2014, 262.

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